NAME _____ DATE _____

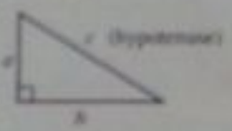
11–6 The Pythagorean Theorem

Objective: To use the Pythagorean theorem and its converse to solve geometric problems.

Theorems

Pythagorean Theorem In any right triangle, the square of the length of the hypotenuse equals the sum of the squares of the lengths of the legs.

$a^2 + b^2 = c^2$

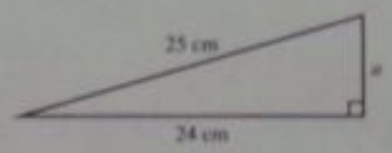


Converse of the Pythagorean Theorem If the sum of the squares of the lengths of the two shorter sides of a triangle is equal to the square of the length of the longest, then the triangle is a right triangle. The right angle is opposite the longest side.

Example 1 The length of one side of a right triangle is 24 cm. The length of the hypotenuse is 25 cm. Write and solve an equation to find the unknown side.

Solution

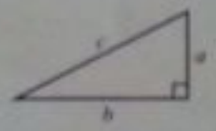
$a^2 + b^2 = c^2$
$a^2 = c^2 - b^2$
$a = \sqrt{c^2 - b^2}$
$= \sqrt{25^2 - 24^2}$
$= \sqrt{625 - 576}$
$= \sqrt{49}$
$= 7$



Check: $7^2 + 24^2 = 25^2$
$49 + 576 = 625$ √

The length of the third side of the right triangle is 7 cm.

In Exercises 1–10, refer to the triangle at the right. Find the missing length correct to the nearest hundredth. A calculator may be helpful.



1. $a = 12, b = 16, c =$? **20.00**
2. $a = 14, b = 48, c =$? **50.00**
3. $a = 7, b = 4, c =$? **8.06**
4. $a = 12, b = 8, c =$? **14.42**
5. $a = 10, b = 10, c =$? **14.14**
6. $a = 14, b = 7, c =$? **15.65**
7. $a =$?, $b = 10, c = 16$. **12.49**
8. $a =$?, $b = 35, c = 37$. **12.00**
9. $a = 10, b =$?, $c = 26$. **24.00**
10. $a = 40, b =$?, $c = 41$. **9.00**

NAME _____ DATE _____

Example 2 State whether or not the three given numbers represent the lengths of the sides of a right triangle.

a. 5, 12, 13 b. 12, 18, 22

Solution

a. $a^2 + b^2 = c^2$
$5^2 + 12^2 \stackrel{?}{=} 13^2$
$25 + 144 \stackrel{?}{=} 169$
$169 = 169$ √

5, 12, and 13 form a right triangle.

b. $a^2 + b^2 = c^2$
$12^2 + 18^2 \stackrel{?}{=} 22^2$
$144 + 324 \stackrel{?}{=} 484$
$468 \neq 484$

12, 18, and 22 do not form a right triangle.

State whether or not the three given numbers represent the lengths of the sides of a right triangle.

11. 9, 16, 20 **no**
12. 9, 40, 41 **yes**
13. 9, 12, 15 **yes**
14. 5, 6, 7 **no**
15. 6, 8, 10 **yes**
16. 11, 12, 16 **no**
17. 18, 24, 30 **yes**
18. 45, 60, 75 **yes**
19. 17, 18, 33 **no**
20. 20, 21, 29 **yes**
21. 21, 28, 35 **yes**
22. 15, 36, 38 **no**

In Exercises 23–30, refer to the diagram for Exercises 1–10. Find the missing length correct to the nearest hundredth.

23. $a = b = 16, c =$? **22.63**
24. $a = b = 9, c =$? **12.73**
25. $a = 16, b = \frac{1}{2}a, c =$? **17.89**
26. $a = \frac{1}{2}b, b = 10, c =$? **11.18**
27. $a = 12, b = \frac{1}{3}a, c =$? **12.65**
28. $a = 16, b = \frac{1}{4}a, c =$? **16.49**
29. $a = b = 10, c =$? **14.14**
30. $a = \frac{1}{2}b, b = 8, c =$? **8.94**

Mixed Review Exercises

Simplify.

1. $\sqrt{25x^{12}y^2}$ **$5x^6|y|$**
2. $\sqrt{c^2 - 8c + 16}$ **$|c - 4|$**
3. $\sqrt{48a^5(b+2)^2}$ **$4a^2|b + 2|\sqrt{3a}$**

Write a fraction in simplest form.

4. $(2 \cdot 10^{-2})^3$ **$\frac{1}{125{,}000}$**
5. $(3x^{-2}y^{-3})^2$ **$\frac{9}{x^4y^6}$**
6. $\frac{5}{6} + \frac{6}{5}$ **$\frac{25}{36}$**
7. $\frac{a-2}{4} + \frac{2-3a}{6}$ **$\frac{-3a-2}{12}$**
8. $4y + \frac{y-1}{y-2}$ **$\frac{4y^2 - 7y - 1}{y-2}$**
9. $\frac{3r^2}{5x} + 15rx$ **$\frac{r}{25x^2}$**
10. $\left(-\frac{k^2}{4}\right)^3$ **$-\frac{k^6}{64}$**
11. $\frac{2x^2 - 16x - 40}{2x - 20}$ **$x + 2$**
12. $\frac{2x^2 + 5x - 3}{3 + x}$ **$2x - 1$**